Routing and Permitting Techniques of Overweight Vehicles

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Abstract: Overweight vehicles require permits to cross the highway bridges, which are designed for “design load vehicles” (prescribed in the national standards). A new, fast, and robust method is presented for the verification of bridges, which requires minimal input only: the axle loads, axle spacing, the bridge span(s), and the superstructure type. The bridge can be a single or a multispans girder, an arch bridge, a frame structure, or a box girder. The overweight vehicle may operate within regular traffic or it may cross the bridge at a given lane position while other traffic is prohibited on the bridge. The method is illustrated by numerical examples for deck-girder bridges and for a box girder.

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Introduction

Overweight and oversize vehicles require permits to reach their destinations. To obtain the optimum route an optimization process must be developed (Osegueda et al. 1999; Adams et al. 2002), which includes the verification of the safety of the bridges (Correia and Branco 2006).

To fulfill this goal a detailed structural analysis can be carried out in which every structural element of the bridge must be analyzed. This method is not feasible in most cases because it requires detailed geometric and member material data, which are not available for the designer. To overcome this difficulty three methods were discussed recently (Vigh and Kollár 2006), and are based on a comparison of the effects of the design load vehicles and the overweight vehicles. These are as follows:

• Comparison of the internal forces (e.g., Correia and Branco 2006);
• Comparison of the axle loads (e.g., U.S. Department of Transportation 1994); and
• Application of artificial influence lines.

The bridge originally was designed for a “design load vehicle” (DLV) which is described in the national code, and it may be assumed that the bridge can withstand the internal forces (bending moments, shear forces, reaction forces, etc.) caused by the DLV. The bridge is safe if the internal forces caused by the overweight vehicle (OV) are smaller than those caused by the DLV

\[ E^{OV} \leq E^{DLV} \]

This relationship can be written as

\[ n = \frac{E^{DLV}}{E^{OV}} \geq 1 \]

where \( n \) = safety of the bridge.

These are the basic equations of the “comparison of internal forces” which can be applied if the appropriate bridge parameters are available, (the spans, the relative stiffness for multispans bridges, the geometry of truss girders, etc.).

The “comparison of the axle loads” (e.g., the use of the Federal bridge formulae) does not require the bridge data; however, it can be inaccurate for long span or multispans bridges (Chou et al. 1999; James et al. 1986; Kurt 2000; Ghosn 2000).

As a compromise we proposed and verified a new method, the “application of artificial influence lines,” which requires only the span(s) of the bridge and it is also reliable for long span and multispans bridges, truss, girder, and arch bridges (Vigh and Kollár 2006), and frames (Vigh 2006).

Hence, either the “comparison of internal forces,” if the necessary data are available, or the “application of the artificial influence lines” can be used (Fig. 1) for evaluating the safety of a bridge.

However, neither of these methods addresses the spatial behavior of the bridge. The effect of the OV may be smaller on the bridge than that of the DLV because of the different placement of the load through the width of the bridge (Fig. 2). The spatial effect can be taken into account by numerical finite-element method (FEM) calculations or with the aid of “distribution factors” (Goodrich and Puckett 2000).

The permit for an OV can be issued with the restriction that the OV may cross the bridge at a given lane position and/or prohibit simultaneous loading of other traffic on the bridge. These options were not considered in either of the two recommended methods above.

In this paper, we will address these issues and give a complete algorithm for issuing permits for OVs.

Problem Statement

An OV requires a permit when it crosses a bridge. The safety of the bridge \( n \) loaded by the OV must be determined for the following cases:
When the OV is part of the traffic (additional traffic loads are allowed on the bridge); when the other traffic loads are not permitted (Fig. 3); and when the other traffic loads are not permitted and the OV must go along the middle of the bridge [Fig. 2(c)]. The OV may cross the bridge when $n \geq 1$. Note that in the calculation of $n$ both the possible failure of the main load carrying members and the failure of the deck, cross girders, etc., must be considered; the first will be referred to as “global” failure while the second will be referred to as “local” failure.

**Approach**

To solve the problem defined in the previous section the spatial behavior of the bridge must be considered. For example, the highest internal forces in the exterior girder of a deck-girder bridge is developed when the heavy vehicle is placed close to the exterior of the bridge [Fig. 4(a)], while the area of the deck (not covered by this vehicle) is loaded by the additional traffic, which is usually represented by (uniformly) distributed loads [Fig. 4(a)].

To avoid the spatial analysis bridge designers often apply influence lines with the aid of a (2D) beam model [Fig. 4(b)]. In Fig. 5 typical influence lines of the exterior girder ($\eta_E$) and the middle girder ($\eta_M$) of deck-girder bridges are shown. The ordinates of the influence lines can be used to determine the load on the girders. When the load, $P$, is placed at an arbitrary position through the width, the load on the exterior girder is $P_{E} = \eta_E P$, while the load on the middle girder is $P_{M} = \eta_M P$ [Fig. 5(d)], where $\eta$ = “distribution factor” (Goodrich and Puckett 2000).

A few typical influence lines of the left exterior girders are shown in Fig. 6. The torsional stiffness of the girders was not considered. The influence lines in the left column were determined assuming that the cross girders (or the slab through the width) have very high stiffness while in the calculation of the influence lines in the right column they have finite stiffness. Typical influence lines of the middle girders are shown in Fig. 7, again
assuming infinite and finite stiffness for the cross girders shown in the left and right columns, respectively.

Note that the smaller the ratio of the bridge’s length to width, the more important the effect of the deformation of the cross girders.

Several methods are available in the literature for the calculation of the influence lines, e.g., the Guyon–Massonnet (Guyon 1946; Massonnet 1950) method, which also takes into account the torsional stiffness of the girders.

In the recommended permitting procedure, the spatial effect will be taken into account with the aid of influence lines. To obtain an “accurate” influence line pertinent bridge data are needed that may not be easily obtained. Two important notes are stated below:

First, we use the influence lines only for comparing the DLV and the OV, and hence for the presented procedure, only the shapes of the influence lines are important; and the numerical values of the lines are irrelevant.

Second, if the accurate line is not known, we may replace it by two lines which are upper and lower approximations of the influence line (Fig. 8), one of these (unknown) is a conservative estimation of the influence line.

Note that the more bridge data available the more accurate

the approximation of the influence lines (Goodrich and Puckett 2000).

The minimal input data of the bridge for the permitting procedure are: the span, the width, and the type of superstructure. With the aid of this information, the influence lines can be approximated.

We emphasize that both upper and lower approximations of the influence lines are used (Fig. 8) and as a consequence the recommended solution is conservative. When we have limited knowledge on the superstructure, the upper and lower approximations are very different and the approximation may be too conservative.

**Building Blocks of Procedure**

In this section the steps of the calculation of the permitting process will be given. In the same steps, the designer may apply different approximations and refined procedures; nevertheless, in the following subsection we will present a simple method, which is also implemented in our computer code.

**Upper and Lower Bounds of Influence Lines**

The influence lines can be calculated, e.g., by the finite-element method, by the Guyon–Massonnet procedure, etc. It can be precalculated for each bridge and then be stored in a database. The stored ordinates of the influence lines can be used in the permitting procedure.

When these precalculated influence lines are not available, we recommend the following approximations:

The influence line of the exterior girder $\eta_E$ is approximated by a straight line [Fig. 9(a)]; the ordinate of the left side is equal to one. (Remember that only the shape of the line matters, hence one of the ordinates can be chosen arbitrarily.) The ordinate on the right side depends on the $b/l$ (width to length) ratio and on the superstructure type.

The influence line of the middle girder is approximated by a roof shape function [Fig. 9(b)], the ordinate of the middle is equal to one. The ordinates on the two sides ($\eta_{MR}$) depend on the $b/l$ ratio and on the superstructure type.

The recommended values of $\eta_E$ and $\eta_{MR}$ for different bridge types are listed in Table 1. Note that for each bridge type mini-
mum and maximum values are given. These values were obtained by considering the influence lines of several real bridge superstructures (Jankó 1998). If the bridge cannot be classified according to the categories listed in the first row of Table 1 the lowest and the highest values of each row are used.

The values of Table 1 are approximations. The designer may use other categories with different values, or the influence lines can be calculated numerically.

**Placement of Load on Influence Lines**

In this section we will give the calculation of the loads \( P_\eta \) (see Fig. 4), which includes the spatial behavior of the bridge and also the values of the distributed loads which represent the traffic load in addition to the DLV or OV vehicle.

Four cases will be considered which are important in the permitting procedure as follows:

1. **No additional traffic load is considered.** The bridge is loaded with the DLV and OV. Every influence line must be loaded such that the vehicles cause the maximum effect. One of the influence lines is shown in Fig. 10. The loads which must be taken into account on the 2D calculation (Fig. 4) are determined as follows

\[
P_{\eta_{DLV}} = \frac{p_{DLV}}{2}(\eta_{DLV}^1 + \eta_{DLV}^2), \quad P_{\eta_{OV}} = \frac{p_{OV}}{2}(\eta_{OV}^1 + \eta_{OV}^2)
\]

(1)

This calculation must be performed for all the influence lines considered (four lines in the case of Table 1). [For the influence line shown in Fig. 9(b) one of the tires of the vehicle must be either over the middle point or next to the deck edge, where one of these causes the maximum effect (see Fig. 11).] The **worst case influence line** is the one for which the ratio \( P_{\eta_{DLV}} / P_{\eta_{OV}} = \text{minimum} \)

2. **Additional traffic load with both the DLV and the OV.** In addition to the overweight vehicle, the traffic is represented by a UDL, denoted by \( p \). The axle loads acting on the 2D model are calculated by Eq. (1).

The distributed load can also be calculated with the influence lines. In the case of the influence line given in Fig. 12 the UDLs are calculated as

\[
P_{\eta_{DLV}} = pb_{DLV}\frac{\eta_{DLV}^1 + \eta_{DLV}^2}{2}, \quad P_{\eta_{DLV}} = pb_{DLV}\frac{1 + \eta_{DLV}^2}{2}
\]

(3)

\[
P_{\eta_{DLV}} = pb_{DLV}\frac{\eta_{DLV}^1 + \eta_{DLV}^2}{2}, \quad P_{\eta_{DLV}} = pb_{DLV}\frac{1 + \eta_{DLV}^2}{2}
\]

(4)

where the subscripts “in” and “out” refer to the load along and outside of the vehicle [Fig. 4(b)].

These calculations must be carried out for all the influence lines considered. We choose that influence line for which the effect of the OV is the highest, together with the corresponding distributed loads.

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**Table 1. Ordinates of Approximate Influence Lines (Fig. 9) for Different Bridge Structures**

<table>
<thead>
<tr>
<th>Type of bridge</th>
<th>Two-girder system</th>
<th>Multiple-girder system, solid slab</th>
<th>Box</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta_{ER} )</td>
<td>( l \geq b )</td>
<td>( 1/4 )</td>
<td>( l \leq b )</td>
</tr>
<tr>
<td>( \eta_{MR} )</td>
<td>( l \geq b )</td>
<td>( 0 )</td>
<td>( 1 - b/l )</td>
</tr>
</tbody>
</table>

Note: \( l = \) span; and \( b = \) width of bridge.

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Additional traffic load with the DLV only; the highest effect

\[ \sum p_{DLV}^\eta + p_{DLV}^\nu L_{DLV} + p_{DLV}^\sigma + p_{DLV}^\tau \rightarrow \text{minimum} \quad (5) \]

The summation must be carried out for all the axles, while
\( L_{DLV} \), \( P_{DLV} \), \( p_{DLV}^\nu \), and \( p_{DLV}^\sigma \) = lengths of the distributed loads considered. (See the end of this section.)

In the following calculations only the \( p_{DLV}^\eta \) and \( p_{DLV}^\nu \) will be considered for which Eq. (5) is minimum. These loads are denoted by
\[ p_{DLV}^\eta = P_{DLV}, p_{DLV}^\nu = P_{DLV}. \]

In many cases the heavy vehicle governs expression (5), and hence the same influence line is obtained as in Case 1.

Cases 3 and 4 represent the situation when the additional traffic is not allowed together with the OV. Both cases can be calculated with two options: (1) the OV must go along the middle of the bridge; and (2) the OV may be placed anywhere along the width of the bridge. (This option was also used in Cases 1 and 2.) Depending on the length and the width of the bridge, two different cases may govern the permitting process, which are considered below under Cases 3 and 4.

3. Additional traffic load with the DLV only; the highest effect of OV. The calculation of the \( P_{DLV}^\eta \) and \( P_{DLV}^\nu \) can be performed by Eq. (1), but in the case of “Option 1,” the ordinates \( \eta_1 \) and \( \eta_2 \) are calculated according to the middle position of the OV, as shown in Fig. 13.

The distributed load together with the DLV is calculated by Eq. (3), while there are no distributed loads together with the OV.

The condition to choose the worst case influence line is identical to Eq. (5), when the terms \( p_{DLV}^\eta \) and \( p_{DLV}^\nu \) are set equal to zero

In the following calculations only those \( p_{DLV}^\eta \) and \( p_{DLV}^\nu \) will be considered for which Eq. (6) is minimum, while \( p_{DLV}^\eta = p_{DLV}^\nu = 0 \). These loads are denoted by
\[ p_{DLV}^\eta = P_{DLV}^\eta, p_{DLV}^\nu = P_{DLV}^\nu. \]

4. Additional traffic load with the DLV only, the smallest effect of \( P_{DLV}^\eta \). For long bridges, the effect of the additional traffic can be significant because of the large bridge deck area. As a consequence, in this case we have to consider the influence line with the smallest area to obtain the smallest effect of the additional traffic load. The condition to choose the influence line is

\[ \sum p_{DLV}^\eta \rightarrow \text{minimum} \quad (7) \]

In the following calculations, only that influence line will be considered for which Eq. (7) is minimum. The corresponding loads are denoted by \( p_{DLV}^\eta, p_{DLV}^\nu, p_{DLV}^\sigma, p_{DLV}^\tau \), and \( P_{DLV}^\eta \).

Using the above cases the spatial effect can be taken into account.

Case 1 is applicable for a simplified analysis and is based on the assumption that the effect of the additional traffic load is the same for the DLV and for the OV. This is a conservative assumption, when the occupied area of the DLV is smaller than that of the OV.

Case 2 can always be used instead of Case 1, and usually gives similar answers.

Case 3 typically results in the influence line shown in Fig. 14(a), because in this case the effect of the OV is rarely
affected by the placement of the OV at the middle. Case 4 may likely result in the influence line shown in Fig. 14(b), where the area is small.

In expressions (5) and (6) the lengths of the loads \( p_{\text{in}} \) and \( p_{\text{out}} \) (Fig. 4), denoted by \( L_{\text{in}} \) and \( L_{\text{out}} \) must be used. The simplest choice of \( L_{\text{in}} = \) length of the vehicle, while that of \( L_{\text{out}} \) can be the difference of the bridge span and the length of the vehicle. This may be unrealistic if the span is small, especially if it is smaller than the length of the vehicle. In expressions (5) and (6) the nominator and denominator represent the effect of the OV and the DLV, respectively. These effects can be measured effectively with the aid of artificial influence lines \( \eta_{\text{P}}, \eta_{\text{M}}, \) and \( \eta_{\text{B}} \) (Vigh and Kollár 2006), which can be applied for short and long span bridges. This was implemented in our computer program to calculate effective lengths for \( L_{\text{in}} \) and \( L_{\text{out}} \).

Local Failure

The method of “comparison of internal forces” (e.g., bending moments, shear forces, reaction forces, etc.) investigates the global behavior of the bridge; however, local failure may also occur (e.g., punching shear, failure of the cross girders, or secondary load-bearing elements).

Local failure can be caused by a single tire, or by a single axle; however two (or more) axles close to each other can also cause local failure. The artificial influence lines (shown in Fig. 15) are excellent tools to compare the effects of the DLV and the OV. (For example the \( \eta_{\text{P}} \) influence lines gives the sum of the axle forces which are within the distance of \( l_{\text{P}} \).) The details are given in Vigh and Kollár (2006), and it is not reiterated here.

The longer the artificial influence lines that are chosen for a comparison of the DLV and the OV, the more conservative the results that are obtained.

We recommend the values given in Table 2 to be used in the calculation of local effect, which means that for a bridge with a span equals to 20 m, the sums of those concentrated forces are compared with each other which are within 4 m, and the maximum distance where interaction between the axle loads is considered is 8 m.

**Table 2.** Lengths of Artificial Influence Lines \( l_{\text{P}}, l_{\text{M}}, l_{\text{B}} \) (Fig. 15) for Calculation of Local Safety

<table>
<thead>
<tr>
<th>Type</th>
<th>( l_{\text{P}} )</th>
<th>( l_{\text{M}} )</th>
<th>( l_{\text{B}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>( \text{min}(0.2l;4\text{~m}) )</td>
<td>( \text{min}(0.3l;6\text{~m}) )</td>
<td>( \text{min}(0.4l;8\text{~m}) )</td>
</tr>
<tr>
<td>Truss or arch</td>
<td>( l/n )</td>
<td>1.5( l/n )</td>
<td>2( l/n )</td>
</tr>
</tbody>
</table>

Note: \( l \) = length of the bridge; \( n \) = number of cells for truss or arch bridges.

Special care should be taken in the investigation of one axle only which, of course, is the heaviest axle of the vehicle. When the width of the OV is smaller than that of the DLV the tire loads of the OV are closer and hence their position is more dangerous than that of the DLV. This effect can be taken into account by increasing the single axle load of the OV by a factor \( R_{\text{OV}}/R_{\text{DLV}} \), where \( R_{\text{OV}} \) and \( R_{\text{DLV}} \) = reaction force of a unit axle load of the OV and DLV, respectively, on a simply supported beam, in which only one tire of the DLV causes a reaction force (Fig. 16) on the left side.

**Partial Safety Factors, Dynamic Effects, and Actual Load Bearing Capacity**

In a comparison of the DLV and the OV we may use the characteristic (basic) values of the loads, which means that identical partial safety factors and dynamic factors are assumed for the DLV and for the OV.

However, the designer may have to consider the following options as well:

1. The possible changes in the safety factors in the standards;
2. The possible changes in the calculation of the dynamic factor in the standards;
3. The possible reduction in the safety factor of the OV, because the axle loads are accurately measured (Fu and Hag-Elsafi 2000);
4. The possible reduction in the dynamic factor due to the limitation of the speed of the OV; and
5. Increasing or decreasing the DLV for bridges that are stronger or weaker than their design loading.

For example, the Eurocode applies a safety factor of 1.5 for the DLV (CEN 2000); however, we recommend a safety factor of 1.1 for the OV, when the axle loads are verified.

**Algorithm of Proposed Calculation**

Using the building blocks of the previous section the following steps must be carried out for the permitting procedure of an OV:

First, the influence lines of the bridge (either on the basis of Table 1, or with the aid of more accurate methods) must be calculated (Fig. 17, (i)) then with the aid of these lines the loads are determined for the 2D calculation for the four cases.
described above: \( P_{DLV}^{i}, P_{OV}^{i}, P_{DLV}^{2}, P_{DLV}^{2}, P_{DLV}^{3}, P_{DLV}^{3}, P_{DLV}^{4}, P_{DLV}^{4} \), \( P_{in}, P_{out}, P_{OV}^{2}, P_{OV}^{2}, P_{DLV}^{3}, P_{DLV}^{3}, P_{DLV}^{4}, P_{DLV}^{4} \), \( P_{DLV}^{4} \) and \( P_{OV}^{4} \).

Depending on the available data or the required speed, either the “comparison of the internal forces” or the “application of artificial influence lines” can be used (Fig. 1).

In the first case, all the relevant internal forces must be calculated from the DLV and from the OV (DLV and OV), from which the global safety of the bridge is determined

\[
g_{gl} = \min\left(\frac{E_{DLV}}{E_{OV}}\right)
\]

Then the local safety is calculated

\[
g_{loc} = \min\left(n_{i}, n_{j}, n_{k}\right)
\]

The decision can be made whether the OV is permitted or not to cross the bridge (Fig. 17).

When the second method, “application of artificial influence lines,” is chosen, the procedure given in Vigh and Kollár (2006) is used. The final decision can be made based on the results of the approximate and local analysis. The block diagram is given in Fig. 17.

**Special Consideration**

Some of the standards prescribe more than one load case (DLV). This can be included in the procedure given in the previous sections if the following modifications are made:

1. Cases 1–4 must be calculated \( n_{DLV} \) times, where \( n_{DLV} \) = number of the DLV;

Fig. 17. Block diagram of procedure

Fig. 18. Possible placement of UDL, when it varies with lanes (e.g., in case of Eurocode)
2. In the “comparison of the internal forces” $E_{DLV}$ must be calculated as the envelope of the internal forces of the $n_{DLV}$ design load vehicles; and
3. In the “comparison of artificial influence lines” and the calculation of the local safety, $E_P^{DLV}$, $E_M^{DLV}$, and $E_B^{DLV}$ must be calculated as the envelope of $E_s$ of the $n_{DLV}$ design load vehicles.

Some of the standards [e.g., the Ontario Highway Bridge Design Code (Bakht and Jaeger 1984) and the Eurocode], prescribe different UDLs in the different lanes of the bridge, while the assumed number of lanes depends on the width of the bridge [Fig. 18(a)]. In this case only the load calculation on the 2D model (step ② in Fig. 17) must be modified, however more than one load case must be taken into account, as illustrated for three lanes in Figs. 18(a and b).

### Numerical Examples

We consider single span bridges with different lengths and superstructures. The DLV is shown in Fig. 19(a), where its weight is 800 kN and the gauge is 2.7 m. The weight of the OV is 1,053 kN and the gauge is 3.5 m, [Fig. 19(b)]. The UDL (additional traffic) is 3.7 kN/m².

In every single case, we calculate the global safety with the “comparison of internal forces,” the local safety with the aid of artificial influence lines, and also the global safety based on a comparison of internal forces calculated by a commercial FEM software, Axis VM 7.0 (Inter-CAD Kft. 1991–2005). We considered the following load cases:

1. Load 1: $P_{DLV}$ and $P_{OV}$ no additional load;
2. Load 2: $P_{DLV}$, $P_{OV}$ and 3.7 kN/m² UDL;
3. Load 3: $P_{DLV}$ with 3.7 kN/m² UDL and $P_{OV}$ without UDL.

The placement of $P_{DLV}$ and $P_{OV}$ can be arbitrary in all of the above cases. (See “Placement of Load on Influence Lines”); and
4. Load 4: same as Load 3, however $P_{OV}$ is placed along the centerline of the bridge.

#### Example 1

The first example is a simply supported precast prestressed concrete deck-girder bridge. The span is 16.8 m, the width is 9.72 m, and the superstructure consists of nine I-beams. The bridge is loaded by the DLV and the OV. We compute the axial stress for the bottom fiber of the exterior and that of the middle girder at the midspan, and also the reaction forces of these girders. The deformed shape calculated by FEM is shown in Fig. 20, when the load is at one side of the bridge. Table 3 shows the safety factors...
of the methods for five load cases. The calculation shows that our method is always on the safe side. The steps of our method are as follows:

1. The influence lines are determined first (Fig. 17). From Table 1 (multiple-girder system, \( l \geq b \)) we obtain the lines shown in Fig. 21.

2. Cases 1–4 are calculated (Fig. 17), and the results are summarized in Table 4. Cases 1 and 2 are explained in detail below. The width of the OV exceeds the width of the DLV, hence the flattest influence line results in the lowest \( P_{DLV}/P_{OV} \) ratio [see Eq. (2)]. The loads considered in the analysis (Case 1) are: \( P_{DLV} = 800 \text{ kN}/(0.969 + 0.761) = 692 \text{ kN} \) and \( P_{OV} = 1.053 \text{ kN}/(2(0.969 + 0.699)) = 878.3 \text{ kN} \) [see Fig. 22(a)].

   When the distributed loads are also taken into account (Case 2), Eq. (3) results in the influence line shown in Fig. 22(b). The DLV is represented by

\[
P_{DLV}^{\text{in}} = \frac{800 \text{ kN}}{2} (1 + 0.705) = 697.14 \text{ kN}
\]

\[
P_{DLV}^{\text{out}} = 3.7 \left( \frac{4.46 \text{ m} \cdot 0.537 + 0.962}{2} + 1.76 \text{ m} \cdot 0.537 + 0.743}{2} \right) = 16.41 \text{ kN/m}
\]

\[
P_{OV}^{\text{in}} = 3.7 \left( \frac{9.72 \text{ m} \cdot 0.537 + 1}{2} \right) = 27.63 \text{ kN/m}
\]

3. The bending moment and shear force diagrams are calculated. For Case 2 (which is identical to Load 2) the curves are shown in Fig. 23 (left), while the safety of the cross sections are shown in Fig. 24. The safety of the bridge is \( n_{DLV} = 1.55 \).

   When the OV may cross the bridge at an arbitrary posi-

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**Table 4. Calculation of Step 2 (Fig. 17) for Example 1**

<table>
<thead>
<tr>
<th>Case</th>
<th>Left</th>
<th>Center</th>
<th>Right</th>
<th>( P_{DLV}^{\text{in}} )</th>
<th>( P_{DLV}^{\text{out}} )</th>
<th>( P_{OV}^{\text{in}} )</th>
<th>( P_{OV}^{\text{out}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>0.625</td>
<td>0.250</td>
<td>691.98</td>
<td>0</td>
<td>878.31</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0.537</td>
<td>1</td>
<td>0.537</td>
<td>697.14</td>
<td>16.41</td>
<td>857.44</td>
<td>14.70</td>
</tr>
<tr>
<td>3 Opt. 1</td>
<td>0.537</td>
<td>1</td>
<td>0.537</td>
<td>697.14</td>
<td>16.41</td>
<td>857.44</td>
<td>14.70</td>
</tr>
<tr>
<td>3 Opt. 2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>511.93</td>
<td>0.70</td>
<td>8.99</td>
<td>587.17</td>
</tr>
<tr>
<td>4 Opt. 1</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>511.93</td>
<td>0.70</td>
<td>8.99</td>
<td>587.17</td>
</tr>
<tr>
<td>4 Opt. 2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>511.93</td>
<td>0.70</td>
<td>8.99</td>
<td>587.17</td>
</tr>
</tbody>
</table>
tion and the other traffic is prohibited (Load 3) either Case 3 or Case 4 results in the lowest safety. In this example Case 3 gives (Fig. 23 right) \( n_3 = 1.84 \).

4. The safety against local failure is calculated by the artificial influence lines (Fig. 15), as described in Vigh and Kollár (2006). The results are given in Fig. 25. The lowest value in the intervals \( 0 = x \leq l_B \) is at \( x = 0 \), i.e., a single tire may cause local failure: \( n_l = 200 \text{kN}/2/125 \text{kN}/2 = 1.6 \).

5. The safety of the bridge is governed by bending failure, hence \( n = n_B^2 = 1.55 \) when the additional traffic may also be present on the bridge. When the additional traffic is prohibited local failure governs our calculation and \( n = n_l = 1.6 \).

In Table 3 the results of the FEM calculation are also presented, which gives 13–21% higher safety than the presented method, i.e., the presented method is conservative.

Note that the safety of the bridge can also be calculated by the artificial influence lines (Step 6 in Fig. 17). For this case, the safety are also calculated (see Table 3). The safety for Load 1 can be obtained from Fig. 25 \((n=1.38)\).

**Example 2**

In the second example, we change only the length of the bridge (its span is 33.6 m). Table 5 shows the safety factors of the methods and that of the FEM.

**Example 3**

The last example is a simply supported box-girder bridge. The span is 33.6 m, the width is 9.72 m, and the cross section is shown in Fig. 26. Table 6 shows the safety factors of the methods for the five loadcases.

**Conclusion**

A new method and algorithm were presented to determine the safety of bridges subjected to overweight vehicles. The main advantage of the method is that it requires very few data only: the
span(s) and the width of the bridge; the type of the superstructure; and the axle loads and spacing of the design load vehicle and that of the overweight vehicle; the calculation is fast, and robust; e.g., it can be applied safely for single and multispans girder bridges, arches, frame bridges, and box girders.

It can also take into account that the overweight vehicle may operate within regular traffic or it may cross the bridge at a given lane position while any other traffic load is prohibited on the bridge.

The algorithm can be connected to a bridge databank, and to an optimization algorithm [e.g., the Dijkstra method (Dijkstra 1959)], and thus a fast and reliable tool is obtained to find the proper route of an overweight vehicle and thus permits can be issued automatically.

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### References


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